

Dinámica gravitacional emergente a partir de la acción de un fotón con un índice de refracción efectivo

Emergent gravitational dynamics from a photon action with an effective refractive index

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Resumen

Se investiga un nuevo concepto de la gravitación desde un punto de vista variacional aplicando la Acción obtenida para un fotón desde un sistema de referencia no inercial. Defino un índice de refracción efectivo asociado con el campo gravitacional. Las trayectorias geodésicas obtenidas se corresponden con las de una métrica efectiva. En el límite de campo débil la teoría predice potenciales gravitatorios modificados que pueden explicar las curvas de rotación planas de la galaxia sin recurrir al concepto de energía oscura. En escalas cosmológicas el modelo gravitacional propuesto genera una energía gravitacional efectiva que provoca una expansión acelerada del universo sin necesidad de introducir una constante cosmológica. Posibles consecuencias observables son analizadas.

Palabras clave:

Gravitación; Materia oscura; Energía oscura; Expansión del universo; Agujeros negros

Abstract

I investigate a gravitational framework derived from a variational principle applied to the action of a photon observed from a non-inertial reference frame. We define an effective refractive index associated with the gravitational field. The resulting photon trajectories correspond to geodesic of an effective optical metric. In the weak-field limit the theory predicts a modified gravitational potential that can explain the flat galactic rotation curves without dark matter. At cosmological scales the model generates an effective gravitational energy density that lead to accelerated cosmic expansion without introducing a cosmological constant. Possible observational consequences are discussed

Key words:

Gravitation; Dark matter; Dark energy; Universe expansion; Black holes

Emergent Gravitational Dynamics from a Photon Action with an Effective Refractive Index

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(Dated: March 15, 2026)

We investigate a gravitational framework derived from a variational principle applied to the action of a photon observed from a non-inertial reference frame. The action is expressed as

$$S = \Gamma(r)^2 E_0 n_0 \tilde{t}_0 \quad (1)$$

where $\Gamma(r)$ acts as an effective refractive index associated with the gravitational field. The resulting photon trajectories correspond to geodesics of an effective optical metric. In the weak-field limit the theory predicts a modified gravitational potential proportional to $\sqrt{1/r}$ which naturally produces nearly flat galactic rotation curves without dark matter. At cosmological scales the model generates an effective gravitational energy density evolving as $\rho_g \propto a^{-3/2}$, which leads to accelerated cosmic expansion without introducing a cosmological constant. Possible observational consequences are discussed.

I. INTRODUCTION

The modern description of gravitation is provided by General Relativity, introduced by Einstein in 1915 [1]. This theory has successfully explained a wide variety of phenomena including gravitational lensing, black holes, and gravitational waves [2, 4].

However, several astrophysical and cosmological observations appear to require additional components such as dark matter and dark energy. Galactic rotation curves, for instance, remain approximately flat at large radii, which is inconsistent with the Newtonian expectation based on visible matter alone [3]. Similarly, cosmological observations indicate that the universe is currently undergoing accelerated expansion [2].

These problems have motivated the exploration of alternative gravitational models. Some approaches introduce new matter components, while others modify the laws of gravity themselves [5].

In this work we investigate a different perspective in which gravitational dynamics emerge from the propagation of photons in a medium characterized by an effective refractive index.

II. PHOTON ACTION IN A NON-INERTIAL FRAME

We consider the action of a photon observed from a non-inertial reference frame

$$S = \Gamma(r)^2 E_0 n_0 \tilde{t}_0. \quad (2)$$

The parameters entering this expression are

- E_0 : invariant photon energy measured in an inertial frame
- n_0 : number of discrete temporal transitions
- \tilde{t}_0 : invariant time quantum

The product $n_0 \tilde{t}_0$ corresponds to the elapsed coordinate time

$$n_0 \tilde{t}_0 = \int dt. \quad (3)$$

Thus the action becomes

$$S = E_0 \int \Gamma(r)^2 dt. \quad (4)$$

For photons the relation between coordinate time and arc length is

$$ds = c dt. \quad (5)$$

Therefore

$$S = \frac{E_0}{c} \int \Gamma(r)^2 ds. \quad (6)$$

This expression is formally analogous to Fermat's principle in optics

$$S \propto \int n(r) ds, \quad (7)$$

where the refractive index is

$$n(r) = \Gamma(r)^2. \quad (8)$$

Optical analogies of gravitational fields have been discussed previously in several contexts [2, 4].

III. EFFECTIVE GRAVITATIONAL FUNCTION

We assume that the function $\Gamma(r)$ takes the form

$$\Gamma(r) = \frac{1 + \sqrt{\alpha/r}}{\sqrt{1 - \alpha/r}} \quad (9)$$

where

$$\alpha = \frac{GM}{c^2}. \quad (10)$$

This parameter corresponds to the gravitational length scale associated with a mass M .

IV. VARIATIONAL PRINCIPLE

The photon trajectory is obtained from the stationary action condition

$$\delta S = 0. \quad (11)$$

The corresponding Lagrangian can be written as

$$L = \Gamma(r)^2 \sqrt{\dot{x}^2 + \dot{y}^2 + \dot{z}^2}. \quad (12)$$

Applying the Euler-Lagrange equations

$$\frac{d}{d\lambda} \left(\frac{\partial L}{\partial \dot{x}^i} \right) - \frac{\partial L}{\partial x^i} = 0 \quad (13)$$

leads to photon trajectories equivalent to null geodesics in an effective optical metric.

V. EFFECTIVE SPACE-TIME METRIC

The variational formulation leads to the metric

$$ds^2 = -\frac{c^2}{\Gamma(r)^4} dt^2 + dr^2 + r^2 d\Omega^2. \quad (14)$$

Substituting $\Gamma(r)$ yields

$$ds^2 = -c^2 \left(\frac{1 - \frac{GM}{c^2 r}}{\left(1 + \sqrt{\frac{GM}{c^2 r}}\right)^4} \right) dt^2 + dr^2 + r^2 d\Omega^2. \quad (15)$$

The horizon appears when

$$r_h = \frac{GM}{c^2}. \quad (16)$$

This differs from the Schwarzschild horizon, which occurs at $2GM/c^2$ in General Relativity [2].

VI. WEAK FIELD EXPANSION

In the limit

$$r \gg \frac{GM}{c^2} \quad (17)$$

we can expand

$$\Gamma(r) \approx 1 + \sqrt{\frac{GM}{c^2 r}} + \frac{GM}{2c^2 r}. \quad (18)$$

The corresponding gravitational potential becomes

$$\Phi(r) \approx -2c^2 \sqrt{\frac{GM}{c^2 r}} - \frac{7GM}{2r}. \quad (19)$$

The dominant correction therefore scales as

$$\Phi(r) \propto -\frac{1}{\sqrt{r}}. \quad (20)$$

VII. GALACTIC ROTATION CURVES

The gravitational acceleration is

$$g(r) = -\frac{d\Phi}{dr}. \quad (21)$$

Using the potential derived above we obtain

$$g(r) \propto \frac{\sqrt{M}}{r^{3/2}}. \quad (22)$$

For circular orbits

$$\frac{v^2}{r} = g(r). \quad (23)$$

Thus

$$v \propto M^{1/4} r^{-1/4}. \quad (24)$$

This leads to

$$v^4 \propto M, \quad (25)$$

which resembles the observed Tully–Fisher relation [3].

VIII. COSMOLOGICAL CONSEQUENCES

The gravitational field generates an effective energy density

$$\rho_g \propto a^{-3/2}. \quad (26)$$

The modified Friedmann equation becomes

$$H^2 = H_0^2 \left[\Omega_r a^{-4} + \Omega_m a^{-3} + \Omega_g a^{-3/2} \right]. \quad (27)$$

This corresponds to an effective equation-of-state parameter

$$w = -\frac{1}{2}. \quad (28)$$

Since $w < -1/3$, the model predicts accelerated cosmic expansion without a cosmological constant [2].

IX. CURVATURE PROPERTIES

For a metric of the form

$$ds^2 = -A(r)c^2 dt^2 + dr^2 + r^2 d\Omega^2 \quad (29)$$

the Ricci scalar can be written as

$$R = -A'' - \frac{4}{r}A' - \frac{2}{r^2}(A - 1). \quad (30)$$

Using the weak-field approximation we obtain

$$R \approx \frac{6}{r^{5/2}} \sqrt{\frac{GM}{c^2}} + \frac{12GM}{c^2 r^3}. \quad (31)$$

Unlike the Schwarzschild vacuum solution where $R = 0$, the present model predicts a non-zero curvature outside the central mass [4].

X. PHOTON SPHERE

Circular photon orbits satisfy

$$\frac{d}{dr} \left(\frac{A(r)}{r^2} \right) = 0. \quad (32)$$

Solving approximately gives

$$r_{ph} \approx 2.4 \frac{GM}{c^2}. \quad (33)$$

This is slightly smaller than the Schwarzschild photon sphere

$$r_{ph}^{GR} = 3 \frac{GM}{c^2}. \quad (34)$$

Such deviations could potentially be tested through black hole shadow observations.

XI. DISCUSSION

The proposed model suggests that gravitational phenomena may emerge from an optical-like description of photon propagation in a medium characterized by an effective refractive index.

The theory predicts several observable effects:

- modified gravitational potential
- nearly flat galactic rotation curves
- accelerated cosmological expansion
- modified photon sphere radius

Further work should investigate gravitational lensing predictions and compatibility with cosmological observations.

XII. CONCLUSION

We have explored a gravitational framework derived from a photon action defined in a non-inertial reference frame. The resulting theory produces modified gravitational dynamics capable of explaining several astrophysical and cosmological phenomena without introducing dark matter or dark energy.

Future work should include detailed numerical simulations and observational comparisons.

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